Improved Bounds on the Competetitive Ratio for Symmetric Rendezvous-on-the-Line with Unkown Initial Distancee

some more WIP by Guillaume, Khai Van, Martin and Max

12.11.2020

COGA Research Seminar

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#### Rendezvous-on-the-Line w/ Unknown Distance

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They can each distinguish "forwards" and "backwards" but do not necessarily share the same orientation.

They move over the line with speed 1 and want to minimize their expected meeting time. (more specifically: the competitive ratio)

#### Symmetric Rendezvous-on-the-Line w/ Unknown Distance

A pure movement strategy is a Lipschitz-continuous function  $f : \mathbb{R}_{\geq 0} \to \mathbb{R}$  with f(0) = 0 and  $|f(x) - f(y)| \le |x - y|$ .

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Our goal is to give bounds on the best possible competitive ratio.

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### Recap

# Baston and Gal26,66Ozsoyeller, Beveridge and Isler24,84

Cowpath

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4,591

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# Recap

Baston and Gal	26,66
Ozsoyeller, Beveridge and Isler	24,84
Zig-Zag	20.86
Biased Zig-Zag	19,98
"Markov" Zig-Zag	18,96
"Markov" Zig	17,48
"Markov" Zig with Magnets	16,84
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2D-Rendezvous with $l_1$ -norm	9,182
Cowpath	4,591

### **Research Questions**

Quantify how "zig-zag" almost smooths over the worst-case

- Prove an upper bounds for one of those strategies
- Find a simpler 17-competitive strategy

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Fix scaling factor  $\alpha$ .

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During iteration  $i\in\mathbb{Z}$  a robot moves to position  $\pm\alpha^i$  and then back to 0, i.e.

$$f\left(\left(\sum_{k=-\infty}^{i-1} 2 \cdot \alpha^k\right) + \alpha^i\right) = \pm \alpha^i$$
$$f\left(\left(\sum_{k=-\infty}^{i-1} 2 \cdot \alpha^k\right)\right) = 0$$

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f is Lipschitz-contiuous at 0.

Choose between  $\alpha^i$  and  $-\alpha^i$  with a fair coin flip.

#### Fix scaling factor $\alpha$ and switching probability ${\it p}$

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During iteration  $i \in \mathbb{Z}$  a robot moves to position  $f_i = \pm \alpha^i$ .

For the next iteration  $f_{i+1} = -\alpha \cdot f_i$  with propability *p*.

#### Biased "Spiral"

# W.l.o.g. $d=\alpha^{-1+\varepsilon}$ , so iteration 0 is the first time the robots can meet.

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For each iteration starting with 0 we track the fraction of the robots:

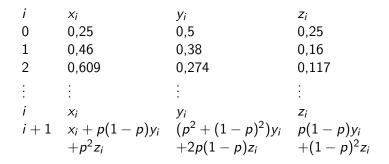
- x) going towards each other (and thus meeting)
- y) running in parallel
- z) going away from each other

i	Xi	Уi	Zi
0	0,25	0,5	0,25
1	0,46	0,38	0,16
2	0,609	0,274	0,117
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$$\bar{X} = \bar{X}t + p(1-p)\bar{Y}t + p^2\bar{Z}t + 0,25$$

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 $ar{Z} = p(1-p)ar{Y}t + (1-p)^2ar{Z}t + ar{1}{4}$ 

$$ar{Y}(t) = rac{rac{1}{4-4(1-p)^2t}+rac{1}{2}}{1-p^2-(1-p)^2t+rac{2p^2(1-p)^2t}{1-(1-p)^2t}}$$

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$$CR(\alpha; p) = \sum_{-\infty}^{0} 2\alpha^{i} + 1 + \alpha \sum_{0}^{\infty} 2y_{i}\alpha^{i} + \alpha \sum_{0}^{\infty} 2z_{i}\alpha^{i}$$

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We can achieve competitive ratio  $CR(1, 16; 0, 6) \approx 26, 33$ 

#### **Proof Pattern**

- 1. Determine possible worst case starting distances
- 2. Compute the stationary distribution of the markov chain
- 3. Determine generating functions for all markov states
- 4. Resolve everything to determine the exact competitive ratio

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During iteration  $i \in \mathbb{Z}$  a robot moves to position  $f_i = \pm \alpha^i$  and stores its last *b* choices.

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For each choice history  $h \in \{-1, +1\}^b$  we go to  $f_{i+1} = -\alpha \cdot f_i$  with propability  $p_h$ .

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Fix all "choices" in advance.



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For consecutive visits to the same side, "bend" the path outwards.

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#### Markov Spiral

Choosing  $\alpha = 1,205$ , b = 5 and

$$p = (1.00, 1.00, 1.00, 1.00, 1.00, 0.59, 1.00, 0.00, 1.00, 0.25, 0.78, 0.44, 1.00, 0.09, 0.53, 0.00, 1.00, 0.00, 0.42, 1.00, 1.00, 0.31, 1.00, 0.00, 1.00, 0.29, 0.33, 0.50, 1.00, 0.59, 0.00, 0.00)$$

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yields a 17, 48-competitive strategy.

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