Improved Bounds on the Competetitive Ratio for Symmetric Rendezvous-on-the-Line with Unkown Initial Distancee

some more WIP by Guillaume, Khai Van, Martin and Max

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COGA Research Seminar

Rendezvous-on-the-Line w/ Unknown Distance

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They move over the line with speed 1 and want to minimize their expected meeting time. (more specifically: the competitive ratio)

Symmetric Rendezvous-on-the-Line w/ Unknown Distance

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Our goal is to give bounds on the best possible competitive ratio.

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- Quantify how "zig-zag" almost smooths over the worst-case
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Fix scaling factor α.

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During iteration $i \in \mathbb{Z}$ a robot moves to position $\pm\alpha^i$ and then back to 0, i.e.

$$f\left(\left(\sum_{k=-\infty}^{i-1} 2 \cdot \alpha^k\right) + \alpha^i\right) = \pm\alpha^i$$

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Choose between α^i and $-\alpha^i$ with a fair coin flip.

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For the next iteration $f_{i+1} = -\alpha \cdot f_i$ with propability p.

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For each iteration starting with 0 we track the fraction of the robots:
x) going towards each other (and thus meeting)
y) running in parallel
z) going away from each other

Distribution over Markov States

i	x_i	y_i	z_i
0	0,25	0,5	0,25
1	0,46	0,38	0,16
2	0,609	0,274	0,117
$\vdots$	$\vdots$	$\vdots$	$\vdots$

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$i+1$	$x_i + p(1-p)y_i$ $+p^2 z_i$	$(p^2 + (1-p)^2)y_i$ $+2p(1-p)z_i$	$p(1-p)y_i$ $+(1-p)^2 z_i$

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Define generating functions $\bar{X} := X(t) := \sum_{i=0}^{\infty} x_i t^i$

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$$\bar{X} = \bar{X}t + p(1-p)\bar{Y}t + p^2\bar{Z}t + 0,25$$

Generating Functions

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\begin{aligned}
\bar{X} &= \bar{X}t + & p(1-p)\bar{Y}t + & \quad p^2\bar{Z}t + \frac{1}{4} \\
\bar{Y} &= & (p^2 + (1-p)^2)\bar{Y}t + & \quad 2p(1-p)\bar{Z}t + \frac{1}{2} \\
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\bar{Y}(t) = \frac{\frac{1}{4-4(1-p)^2 t} + \frac{1}{2}}{1 - p^2 - (1-p)^2 t + \frac{2p^2(1-p)^2 t}{1-(1-p)^2 t}}
$$

Computing the Competitive Ratio

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$$\bar{Y}(t) = \sum_{i=0}^{\infty} y_i t^i = \frac{\frac{1}{4-4(1-p)^2 t} + \frac{1}{2}}{1-p^2-(1-p)^2 t + \frac{2p^2(1-p)^2 t}{1-(1-p)^2 t}}$$

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$$CR(\alpha; p) = \sum_{-\infty}^{0} 2\alpha^i + 1 + \alpha \sum_{0}^{\infty} 2y_i\alpha^i + \alpha \sum_{0}^{\infty} 2z_i\alpha^i$$

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We can achieve competitive ratio $CR(1,16; 0,6) \approx 26,33$

Proof Pattern

1. Determine possible worst case starting distances
2. Compute the stationary distribution of the markov chain
3. Determine generating functions for all markov states
4. Resolve everything to determine the exact competitive ratio

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Fix scaling factor α, a memory depth of b bits and markov transition probabilities $p \in [0,1]^b$

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For each choice history $h \in \{-1,+1\}^b$ we go to $f_{i+1} = -\alpha \cdot f_i$ with propability p_h.

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For consecutive visits to the same side, "bend" the path outwards.

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Markov Spiral

Choosing $\alpha = 1,205$, $b = 5$ and

$$
\begin{aligned}
p = (&1.00, 1.00, 1.00, 1.00, 1.00, 0.59, 1.00, 0.00,\\
&1.00, 0.25, 0.78, 0.44, 1.00, 0.09, 0.53, 0.00,\\
&1.00, 0.00, 0.42, 1.00, 1.00, 0.31, 1.00, 0.00,\\
&1.00, 0.29, 0.33, 0.50, 1.00, 0.59, 0.00, 0.00)
\end{aligned}
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yields a $17,48$-competitive strategy.

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